A very small intro to CTMC in phylogenetics

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Botany 2018

In a Bayesian framework

We are always interested in knowing the posterior distribution

$$P(\theta|D) \propto P(D|\theta) P(\theta)$$

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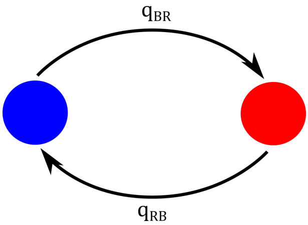

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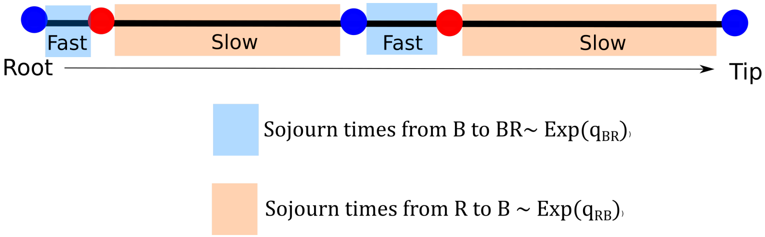
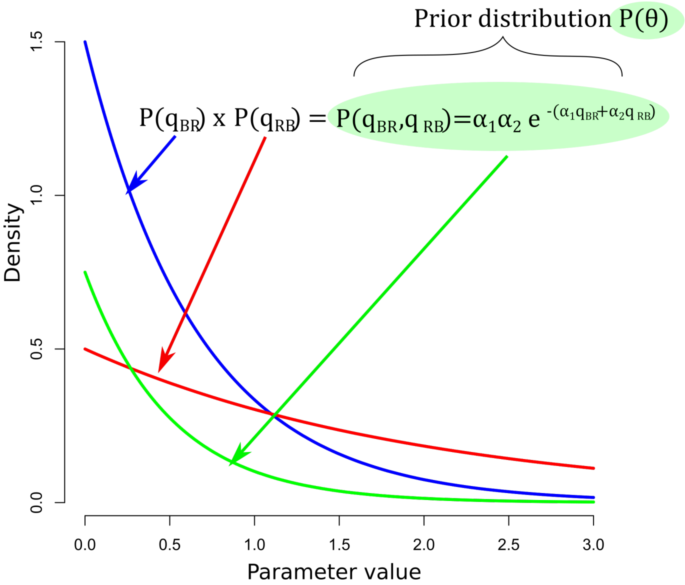


🟦 Sojourn times from B to BR∼ Exp(q_{BR})

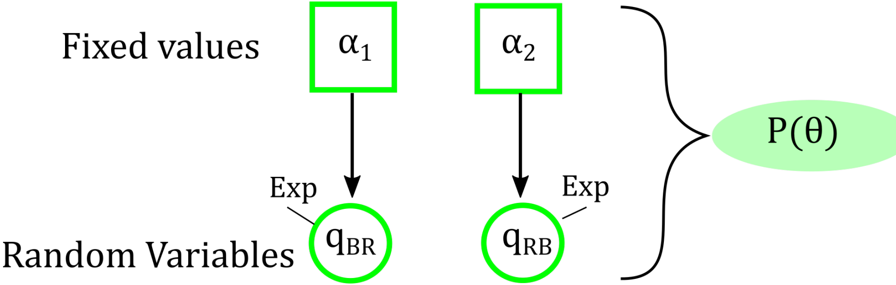
🟧 Sojourn times from R to B ∼ Exp(q_{RB})

The prior distribution: $P(\theta)$



$$P(q_{BR}) \text{ x } P(q_{RB}) = \overbrace{P(q_{BR},q_{RB})=\alpha_1\alpha_2\, e^{-(\alpha_1 q_{BR}+\alpha_2 q_{RB})}}^{\text{Prior distribution } P(\theta)}$$

How are these assumptions represented graphically?



D is our data

We go into our favorite herbarium, field site, or green house and we collect color of multiple species

How do we integrate our model θ and our data D ?

Calculating the likelihood $P(D|\theta)$

- We assume a phylogenetic tree Ψ (for this example is fixed)

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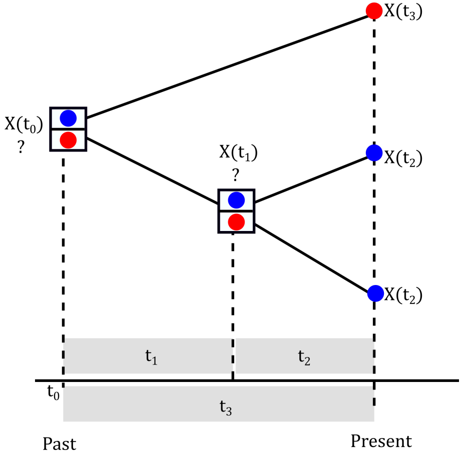
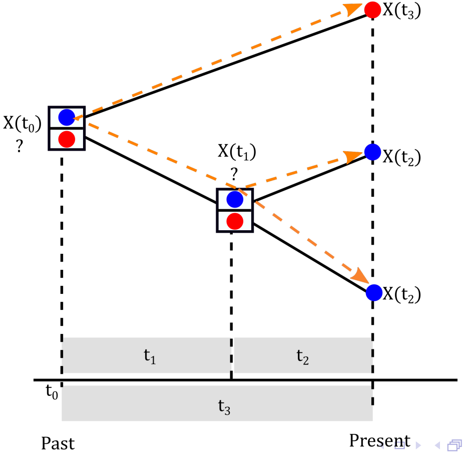
- **Data**: a sample of red and purple flowers on the tips of our phylogeny tree

Likelihood function: The probability of the sample given our hypothesis θ



The probability of a single possible story in phylogenetics

$$P(X(t_2) = B|X(t_1) = B)P(X(t_2) = B|X(t_1) = B)\times$$
$$\times P(X(t_1) = B|X(t_0) = B)P(X(t_3) = R|X(t_0) = B)P(X(t_0) = B)$$



Calculating the likelihood is computationally challenging

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- Reminder: Optimizations to find maximum likelihood estimates and confident intervals require challenging numerical algorithms

How do the rates connect with the probabilities?

Q-matrix= The infinitesimal probability matrix is the derivative of the probability

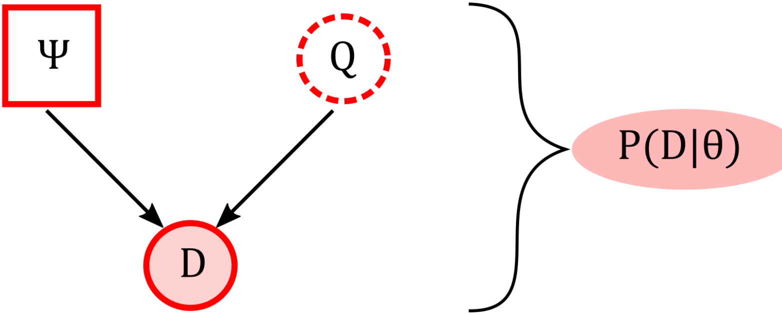
$$\frac{dP(t)}{dt} = Q$$

$$Q= \begin{array}{c c} & \begin{array}{cc} \text{🔵} & \text{🔴} \end{array} \\ \begin{array}{c} \text{🔵} \\ \text{🔴} \end{array} & \begin{pmatrix} -q_{BR} & q_{BR} \\ q_{RB} & -q_{RB} \end{pmatrix} \end{array} \qquad P(t)=e^{Qt}$$

Likelihood in graphical form $P(D|\theta)$

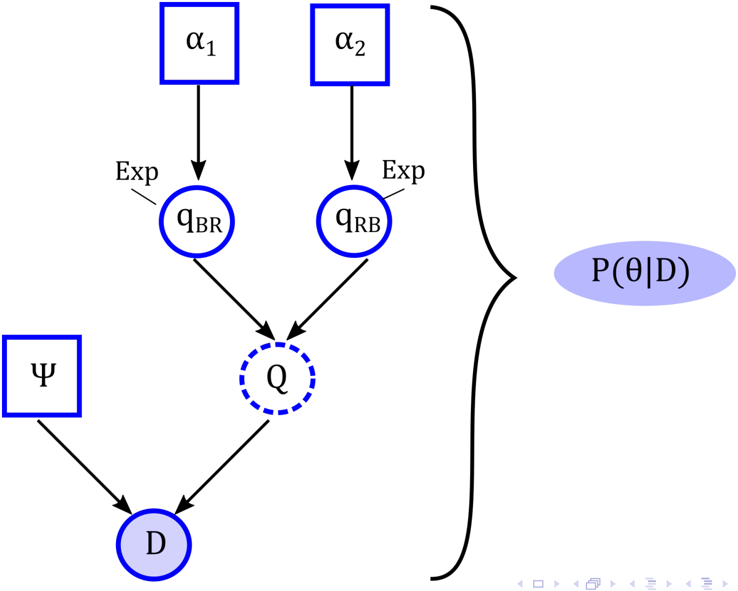
Fixed value

Function calculated from r.v.



Evaluate in observed data.

The posterior distribution: the model conditional to the observed data



Graphical model benefits

- **Explicit notation**: In RevBayes we have notation for fixed variables, random variables, observed data, deterministic function,...

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- **Explicit notation**: In RevBayes we have notation for fixed variables, random variables, observed data, deterministic function,...
- **Modularity**: Once I have built a model I can connect other as a module (building blocks!)